

Blunting the Spike: the CV Minimum Period

A.R. King¹, K. Schenker¹, J.M. Hameury²

1. *Theoretical Astrophysics Group, University of Leicester, Leicester, LE1 7RH, UK*

2. *Observatoire de Strasbourg, UMR 7550 du CNRS, 11 rue de l'Université, 67000 Strasbourg, France*

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ABSTRACT

The standard picture of CV secular evolution predicts a spike in the CV distribution near the observed short-period cutoff $P_0 \simeq 78$ min, which is not observed. We show that an intrinsic spread in minimum (‘bounce’) periods P_b resulting from a genuine difference in some parameter controlling the evolution can remove the spike without smearing the sharpness of the cutoff. The most probable second parameter is different admixtures of magnetic stellar wind braking (at up to 5 times the GR rate) in a small tail of systems, perhaps implying that the donor magnetic field strength at formation is a second parameter specifying CV evolution. We suggest that magnetic braking resumes below the gap with a wide range, being well below the GR rate in most CVs, but significantly above it in a small tail.

Key words: novae, cataclysmic variables — binaries: close — stars: evolution

1 INTRODUCTION

The orbital period P is the one parameter of a cataclysmic variable (CV) which observers can usually measure with confidence. Roche geometry implies a close relation

$$P \propto \left(\frac{R_2^3}{M_2} \right)^{1/2} \quad (1)$$

between this period and the mass and radius M_2, R_2 of the mass-losing star. The accretion luminosity of CVs suggests that the mass transfer timescale $t_M = -M_2/\dot{M}_2$ is considerably shorter than the age of the Galaxy, so that M_2, P change on this timescale. Hence the observed distribution of CV periods is by far the most significant indicator of CV evolution. As is well known, it is consistent with the idea that the evolution is driven by angular momentum losses from the binary orbit (see e.g. King, 1988 for a review).

The observed CV histogram (Fig. 1) cuts off sharply at an orbital period of $P = P_0 \simeq 78$ min. There are respectively 0 and 12 systems in the period ranges $P_0 \pm 5$ min. The idea that P_0 represents a global period minimum ($\dot{P} = 0$ for P_0) for CVs has been widely accepted for the last two decades. It is clear that such a global minimum can exist (Paczynski, 1981; Paczynski & Sienkiewicz, 1981; Rappaport, Joss & Webbink, 1982). As the mass M_2 of an unevolved secondary star in a CV is reduced by mass transfer, the binary period P usually decreases also, with $R_2 \sim M_2 \sim P$. However for very small $M_2 \lesssim 0.1M_\odot$, the secondary's Kelvin–Helmholtz time t_{KH} exceeds the timescale $t_M = -M_2/\dot{M}_2$ for mass transfer driven by gravitational radiation. Instead of shrinking smoothly to the main-sequence radius appropriate to its reduced mass, the star cannot now reduce its entropy quickly

enough and contracts more slowly. From (1) we see that once R_2 decreases more slowly than $M_2^{1/3}$ the orbital period P must begin to increase, defining a minimum (‘bounce’) period P_b for the system. It is important to realize that there is nothing extreme about conditions in the binary at this point; in particular the mass transfer rate remains almost precisely the same ($\sim 4 \times 10^{-11} M_\odot \text{ yr}^{-1}$) as when the system was well above the period minimum, cf Kolb (2002), Fig. 3. Nor do conditions change drastically thereafter: since $M_2 \simeq 0.1M_\odot$ at P_0 we have $t_M \sim 2.5$ Gyr. Hence most CVs do not have time to evolve to significantly longer periods or lower mass transfer rates after reaching P_b : clearly to evolve to $-M_2 = 1 \times 10^{-11} M_\odot \text{ yr}^{-1}$ would require of order the age of the Galaxy. CVs reaching P_0 thus remain clustered there with mass transfer rates similar to those of systems at longer periods.

Detailed calculations always predict a value P_b very close to, if slightly shorter than, the observed P_0 . The discrepancy $P_b < P_0$ is persistent, but may reflect uncertain or over-simple input physics (cf Kolb & Baraffe, 1999). In particular the difference between the true and spherically approximated radii may account for most of the disagreement. However there is a much more serious problem with this interpretation of the observed cutoff at P_0 . This concerns the discovery probability

$$p(P) \propto \frac{(-\dot{M}_2)^\alpha}{|\dot{P}|}. \quad (2)$$

Here α is some (presumably positive) power describing observational selection effects. For example $\alpha = 3/2$ for a bolometric flux-limited sample, while $\alpha = 1$ is often assumed for systems detected via optical outbursts. Since $\dot{P} = 0$ at

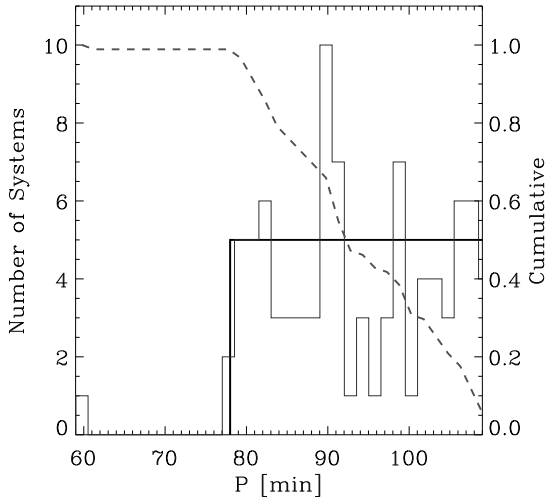


Figure 1. CV period histogram and cumulative distribution (dashed) below the observed period gap (data from Ritter & Kolb, 1998). The Heaviside function shown is the representation (4).

$P = P_b$, $p(P)$ must clearly have a significant maximum there unless $-\dot{M}_2$ declines very sharply near this period. In other words, the observed CV period histogram should show a sharp rise near a global minimum P_b unless the mass transfer rate drops there. However, as mentioned above, all evolutionary calculations show that $-\dot{M}_2$ changes very little as P_b is approached. We conclude that if P_0 is a global minimum there should be a large ‘spike’ in the CV period histogram there (cf Kolb & Baraffe, 1999). Instead, the observed period histogram (Fig. 1) has a ‘corner’ at P_0 , i.e. a sharp cutoff of a fairly flat distribution, but no spike.

The lack of a spike in the observed distribution has prompted numerous theoretical investigations (see Kolb, 2002 for a review, and Barker & Kolb 2002). Most of these propose ways in which CVs might become difficult to discover near P_0 . A basic problem for this type of argument is that, as we have seen, there is nothing at all unusual about the system parameters (mass transfer rate, separation etc) at this period. Further, attempts to use accretion disc properties as a way of making systems hard to discover founder on the fact that the AM Herculis systems, which have no accretion discs, have precisely the same observed short-period cutoff $P_0 \simeq 78$ min, and no spike either. King & Schenker (2002) suggested that CV formation may take roughly the age of the Galaxy, so that the oldest systems have not yet quite reached the minimum period. The fine-tuning here is perhaps worrying, although not easily disproved. In this paper we suggest a solution to the spike problem invoking a different new ingredient in the standard picture of CV evolution.

2 BLUNTING THE SPIKE

The new element we introduce into the standard picture is the idea of a *distribution* of minimum periods P_b . At first sight this may not seem a particularly radical step. However it amounts to allowing the intervention of a second controlling parameter. Second, note that the difficult part here is

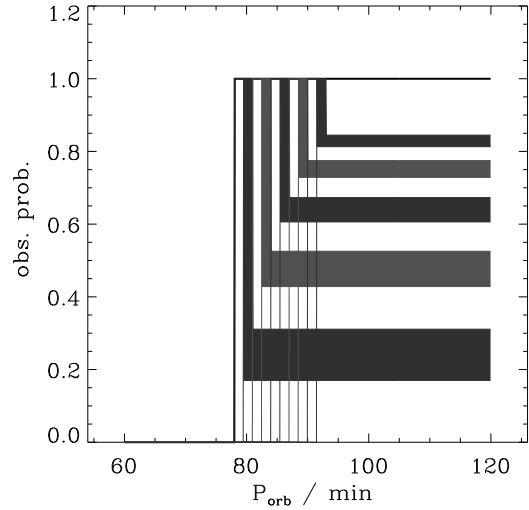


Figure 2. Simple construction of a square distribution by adding discrete L-shaped individual components. The parameters used for the case shown are: $r = 10$, $a = 0.83$, $\Delta P = 1.5$, $P_0 = 78$.

not smearing out the spike, but *simultaneously retaining the sharp cutoff* at P_0 .

To fix ideas we first formulate a simplified version of the spike problem which has the virtue of being exactly solvable, and then proceed to a more realistic approach. In the simplified problem we make the following approximations:

1. We assume that the relative discovery probability for any individual CV depends only on $P - P_b$, and can be represented as

$$p(P - P_b) = H(P - P_b) - aH(P - P_b - \Delta P). \quad (3)$$

Here the H ’s are Heaviside functions, P_b is the bounce period for the individual CV, and $a, \Delta P$ are constants. As can be seen by comparison with the lowest panel of Figure 3 of Kolb (2002) this is a fair representation if we take $a \simeq 0.83$, $\Delta P \simeq 1.5$ min, with $P_b = 67$ min in the case shown there.

2. The observed CV histogram is taken as

$$N(P) = N_0 H(P - P_0) \quad (4)$$

where N_0 is a normalization. This is a relatively crude representation of the observed histogram (Fig. 1), but does capture in essence its only significant feature, the ‘corner’ at P_0 .

Armed with these assumptions, we determine the relative distribution $n(P_b)$ of CVs with minimum period at a given P_b . Clearly the observed histogram $N(P)$ is the convolution of $n(P_b)$ with $p(P - P_b)$, so using (3, 4) we have

$$N(P) = \int_0^P dP_b n(P_b) p(P - P_b) = N_0 H(P - P_0). \quad (5)$$

Taking Laplace transforms, the convolution theorem gives

$$\bar{n}(s) \bar{p}(s) = \frac{N_0}{s} e^{-sP_0}. \quad (6)$$

Now since

$$\begin{aligned} \bar{p}(s) &= \int_0^\infty e^{-sx} [H(x) - aH(x - \Delta P)] dx \\ &= \frac{1}{s} - \frac{a}{s} e^{-\Delta P s} \end{aligned} \quad (7)$$

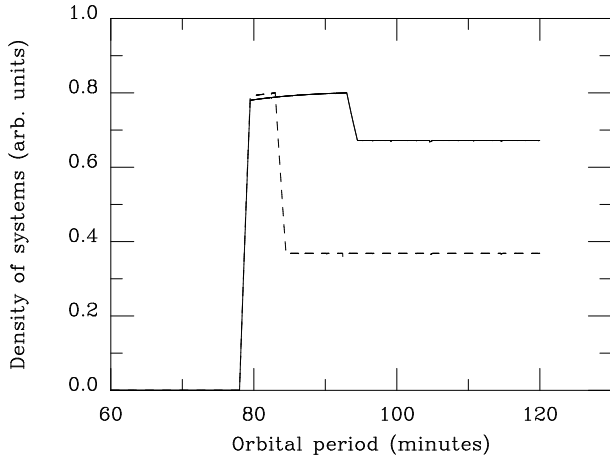


Figure 3. Predicted distribution, assuming (a) that P_b varies over a 15 min range, starting from 78 min, with $n(P_b)$ proportional to $\exp(-k(P_b - P_0)/\Delta P)$, $k = 1.8$, and (b) P_b varies over a 5 min range, with $k = 0.6$. In both cases, the intrinsic width of the spike is 1.5 min.

we get

$$\bar{n}(s) = \frac{N_0 e^{-sP_0}}{1 - a e^{-s\Delta P}} = N_0 \sum_{r=0}^{\infty} a^r e^{-(P_0 + r\Delta P)s}. \quad (8)$$

Hence transforming back we have

$$n(P_b) = N_0 \sum_{r=0}^{\infty} a^r \delta(P_b - P_0 - r\Delta P). \quad (9)$$

This result gives a decomposition of the assumed histogram which is completely obvious when plotted (Fig. 2). However it does illustrate the main point: one can create a rather flat but cut-off distribution (cf 4) from the very spiky individual probabilities (3) by taking a distribution of bounce periods $P_b > P_0$ falling off with P_b (here as $a^{(P_b/P_0)}$) for $P_b > P_0$. Dropping the higher-order terms $r \geq r_{\max}$ produces a downward step at $P = P_0 + r_{\max}\Delta P$ in an otherwise uniform distribution, the discontinuity having size $a^{r_{\max}}$. If a 15% downwards glitch is regarded at the limit of acceptability when compared with observation, we can estimate the spread in P_b required to wash out the spike as $r_{\max}\Delta P = (\log 0.15 / \log 0.83) \times 1.5 \text{ min} = 15 \text{ min}$. Similarly if we define $r_{1/2}$ by $a^{r_{1/2}} = 0.5$ then more than 50% of CVs in the period range $P_0 < P < P_0 + r_{1/2}\Delta P$ are close to their local minima P_b (i.e. are in their local ‘spikes’). We find $r_{1/2} = 3.72$, so we expect half of the CVs in a 5.6 min range above P_0 to be near their local P_b values.

It is now easy to guess a continuous form of $n(P_b)$ which gives a reasonable representation of the corner as $n(P_b) = \exp[-0.124(P_b - P_0)]$. The result is shown in Fig. 3, with the fitting extended to a maximum P_b equal to (a) $P_0 + 5 \text{ min}$, and (b) $P_0 + 15 \text{ min}$. As can be seen, extending to the longer period produces a much more acceptable representation.

We can thus state our main result: the CV distribution near the short-period cutoff P_0 is well reproduced if CVs have a declining distribution of individual minimum periods P_b above the floor value P_0 .

3 THE HIDDEN PARAMETER

In standard CV evolution (e.g. King, 1988) the global properties of a system at any epoch are specified by two parameters, e.g. the initial primary and secondary masses M_{1i}, M_{2i} , corresponding to a white dwarf and an unevolved low-mass star respectively. Given these two quantities we can in principle calculate all others such as period, mean mass transfer rate and discovery probability $P, -\dot{M}_2, p(P)$ at any subsequent time. In practice evolutions starting from different initial secondary masses M_{2i} converge very rapidly to an effectively common track (Stehle et al., 1996), and even the dependence on white dwarf mass is very weak. Thus in this standard picture all global properties of a CV are essentially specified if we know the orbital period P . This extreme simplicity gives the standard picture its predictive power, but also makes it vulnerable to problems such as the missing spike at P_0 . This is clearly predicted if every CV follows the common evolutionary track. To blunt the spike in the manner considered above requires sensitivity to a further physical parameter. Paczyński & Sienkiewicz (1983) show that this cannot be the primary mass or the assumed mean surface opacities (in any case the latter do not in principle constitute an independent parameter).

We therefore turn to the other effect altering P_b considered by Paczyński & Sienkiewicz (1983), namely increasing systemic angular momentum losses \dot{J} by a factor f above the gravitational radiation value \dot{J}_{GR} . They found a $\sim 10 \text{ min}$ spread in P_b for a range $1 \leq f \lesssim 2$. We checked this using the codes of Mazzitelli (1989) – as adapted by Kolb & Ritter (1992) – and Hameury (1991), and found that the required $\gtrsim 15 \text{ min}$ spread results for $1 \leq f \lesssim 6$.

Note that in fixing this range of f we assume that evolution with $f = 1$, i.e. driven purely by GR, does produce the minimum bounce period $P_b = P_0 \simeq 78 \text{ min}$, and thus that theoretical efforts in this direction will succeed. We could instead in principle use a value $f_{\min} > 1$ to bring P_b and P_0 into agreement, but this would rob us of the natural explanation for such a global minimum f_{\min} , namely that this is the limit in which all other angular momentum loss mechanisms are small compared with GR.

4 DISCUSSION

We have shown that the corner in the CV period distribution is reasonably well reproduced provided that CV evolution has a second controlling parameter spreading the minimum periods of an exponential tail of CVs by $\gtrsim 15 \text{ min}$. The most likely candidate for this parameter is an increase of the orbital angular momentum loss rate above that provided by gravitational radiation by a factor f , ranging from 1 to at least 6. Since gravitational radiation must always be present, the requirement $f \geq 1$ simply implies another angular momentum loss process whose strength ranges from values $\ll |\dot{J}_{\text{GR}}|$ to $\gtrsim 5|\dot{J}_{\text{GR}}|$. An obvious candidate here is varying admixtures of magnetic stellar wind braking. The physical origin of the extra degree of freedom specified by f might then simply be the intrinsic strength of the stellar magnetic field in the region of the ISM from which the secondary star formed, which may then influence the strength of any dynamo-amplified or generated field. It is an ob-

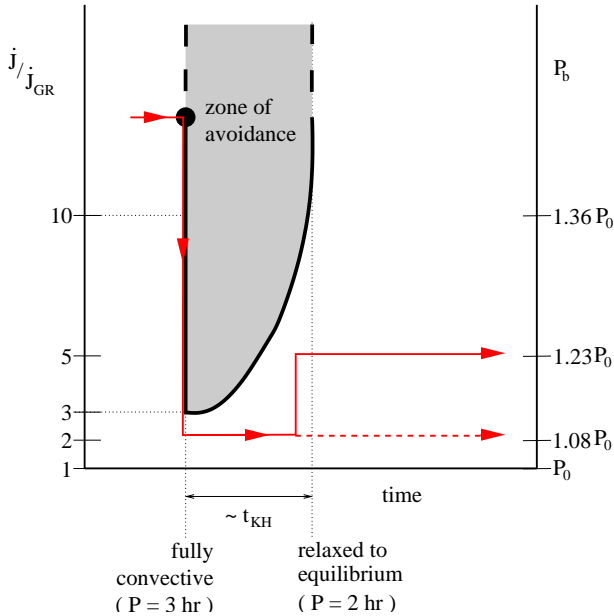


Figure 4. Forbidden zone (shaded area) for angular momentum losses \dot{J} in the interrupted magnetic braking picture (Rappaport, Verbunt & Joss, 1983, Spruit & Ritter, 1983) illustrated schematically for a CV becoming fully convective at $P \simeq 3$ hr with magnetic braking rate $\dot{J}_0 \gg 10 \dot{J}_{GR}$. To form the observed period gap between ~ 3 and ~ 2 hours, \dot{J} must drop rapidly from a value of \dot{J}_0 to $\lesssim 3 \dot{J}_{GR}$, and stay low for a Kelvin time. After this, any amount of magnetic braking may be possible, leading to significantly different bounce periods P_b as given on the righthand axis.

served fact that otherwise similar stars can have very different magnetic field strengths, and this presumably applies to the secondary stars of CVs also. Apparently in most cases this field gives only weak magnetic braking $|\dot{J}_{MB}| \ll |\dot{J}_{GR}|$, but there is evidently a tail of systems with stronger braking. This would for example naturally result if the distribution of intrinsic magnetic fields were a gaussian, with the mean sufficiently high to give strong braking above the period gap, but too low for such braking below the gap in all but an exponential tail of CVs.

This is a satisfying conclusion, as one of the objections to the usual mechanism for forming the CV period gap between 3 – 2 hours has been that strong magnetic activity is observed in main-sequence stars with masses low enough to be secondaries in CVs below the gap. The answer to this objection is now that indeed such activity may well occur in systems below the gap, but in most cases the resulting braking is weaker than GR. However for $f \sim 6$ much of the tail has values of magnetic braking considerably stronger than the extrapolation of the usual magnetic braking laws adopted for CVs above the gap (Verbunt & Zwaan, 1981 Mestel & Spruit, 1987). Evidently these laws do not describe this regime well, in agreement with the idea that the nature of the braking changes character once the secondaries become fully convective (Taam & Spruit, 1989). It is therefore also plausible that intrinsically stronger fields are required below the period gap in order to give significant braking rates, thus accounting for the exponential tail of systems with strong braking as suggested above.

We note that gap formation requires only that mag-

netic braking \dot{J}_{MB} should be *interrupted*, not *suppressed*, at $P \simeq 3$ hr (plausibly where the secondary first becomes fully convective). Figure 4 illustrates cases where the braking after the detached phase almost reaches the value before it. Provided magnetic braking does not resume, at something like its former strength, before the oversized secondary has managed to shrink significantly towards its main-sequence radius (on a timescale $\sim t_{KH} \sim 10^8$ yr), a gap of the observed properties will result. If the association with the change to a fully convective structure is maintained, we would thus require that the resulting change in field topology or strength takes at least a time $\sim t_{KH}$. To achieve the required smearing in P_b the angular momentum losses of a fraction of CVs must change in a way similar to the curve shown in Fig. 4.

This discussion suggests that there is no obvious reason for any upper limit on the factor f , given only that the distribution of f values drops as f increases above unity. Systems with large f have high mass transfer rates, and bounce at quite long periods. This is perfectly consistent with observation, provided that only a minority of CVs below the gap have such high values of $|\dot{J}_{MB}|$. We thus suggest that magnetic braking has the following properties (cf Fig. 4)

(i) in most CVs above the period gap it drives mass transfer rates $-\dot{M}_2 \gtrsim 5 \times 10^{-9} M_\odot \text{ yr}^{-1}$

(ii) when a CV secondary becomes fully convective, magnetic braking rapidly drops to very low values for at least the thermal timescale of the star

(iii) magnetic braking is present in all CVs below the period gap, and its strength depends on an inherent property of the secondary, possibly the formation magnetic field. In most cases it is weaker than gravitational radiation, but there is an exponential tail of systems with much stronger braking. These values are much bigger than given by extrapolating the usual magnetic braking laws to short orbital periods, and may thus require the strongest fields.

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